Information, bankruptcy and welfare

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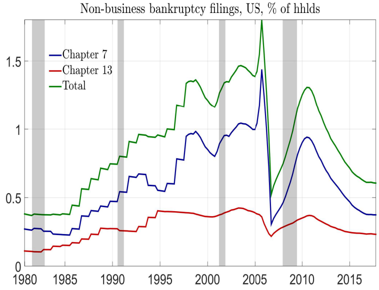
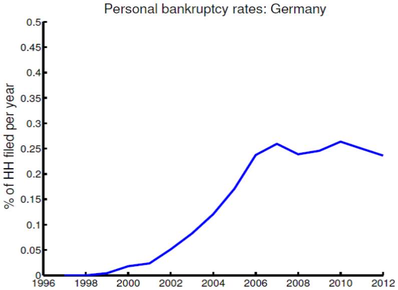
Outline

Motivation



Source: U.S. Courts



Source: Exler (2017)

Motivation

- Increase in debt/bankruptcy lead to reforms (US, IE, DE, etc.)
- Heterogeneity in legislation
- More lending in countries with forgetfulness/forgiveness.
- Trade-off btw risk sharing and price effects - holding Y constant
  - Present and past information, how does it affect this trade-off?
  - Maturity of debt, how does it affect this trade-off?

This presentation: analyse utilitarian welfare in a HACT framework with bankruptcy under different information structures and debt maturities

Results

We study 3 bank information regimes in a HACT framework

1. No information (NI) $\rightarrow$ only distinguish borrowers and lenders
2. Limited information (LI) $\rightarrow$ observe income type
3. Full information (FI) $\rightarrow$ (2) + asset position

Findings (preliminary!):

- Bankruptcy may improve welfare
- This depends on the information structure
- The debt limit may endogenously become tighter than the NBL
  $\rightarrow$ Bankruptcy can take us to the optimal debt limit
- There is more lending with less information

Brief comment on literature

- Zame (1993), Dobbie and Song (2015) and Li and Sarte (2006)
- Athreya (2002), Mateos-Planas and Seccia (2005),Chatterjee et al. (2007) and Exler (2016)
- Elul and Gottardi (2011), Bhaskar and Thomas (2017) and Kobvasyuk and Spagnolo (2017)
- Chatterjee and Eyigungor (2012)
- Nuno and Thomas (2017)

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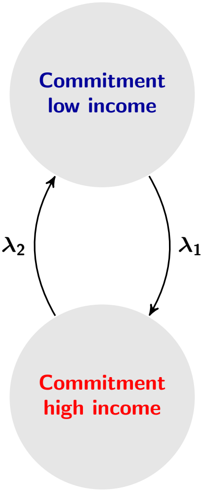
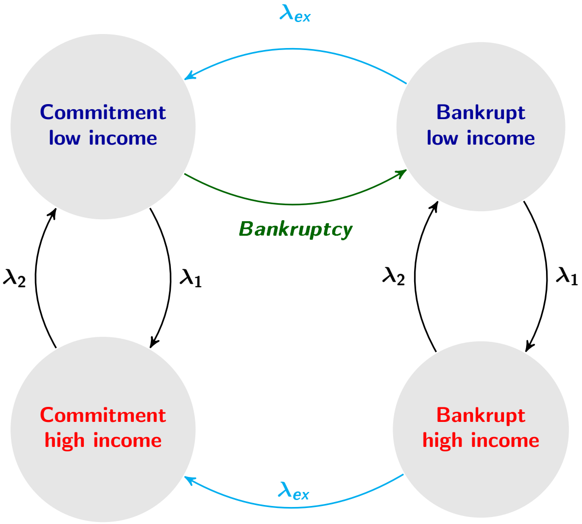
2. Model
   - Households
   - Banks and debt
   - Recipe for equilibrium
3. Partial equilibrium - intuition
   - Consumption and savings
   - Wealth distribution
   - Bond prices
4. General equilibrium results
   - Bankruptcy + more info = higher welfare??
   - The effect of memory
5. Conclusion

Visual representation - Bayer and Wälde (2010) - Achdou et al. (2017)



Visual representation - this model



Households in commitment

Agents maximise utility subject to a flow budget constraint

$$V_i^C = \max_{c,T} E_t \left[\int_t^\infty e^{-\rho(s-t)} u(c_i) \mathrm{d}s + e^{-\rho(T-t)} \left(\hat{V_i^D} - \psi_D - V_i^C \right) \right]$$

$$\text{s.t.} \quad \frac{\mathrm{d}a}{\mathrm{d}t} = a^{\text{new}} - \delta_A a \rightarrow \frac{\mathrm{d}a}{\mathrm{d}t} = \frac{[r + \delta_A]a + z_i - c}{Q_i^C} - \delta_A a$$

- There is an exogenous debt limit (set very loose)
- CRRA utility function
- Income jumps from z_L to z_H and vice versa

Households in bankruptcy

Agents maximise utility subject to new constraints

$$V_i^D = \max_c E_t \left[\int_t^\infty e^{-\rho(s-t)} u(c_i) \mathrm{d}s \right]$$

$$\text{s.t.} \quad \frac{\mathrm{d}a}{\mathrm{d}t} = \frac{[r+\delta_A]a + z_i - c}{Q_i^D} - \delta_A a$$

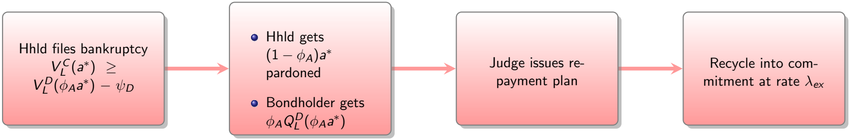
$$\text{s.t.} \quad \frac{\mathrm{d}a}{\mathrm{d}t} \geq \psi_1 + \psi_2 |a|^{\psi_3}$$

$$\text{s.t.} \quad a \geq 0 \quad \text{(after repayment)}$$

- At rate λ_{ex} bankruptcy is forgotten
- Repayment occurs first, on average
- Income still jumps from z_L to z_H and vice versa

Household HJBs - KFE

Bankruptcy scenario



Bankruptcy regime governed by:

- Repayment plan - calibrated as in CH13
- λ_{ex} - following literature
- High income type cannot file
- ψ_D - non-monetary cost - free

Banks, debt and information

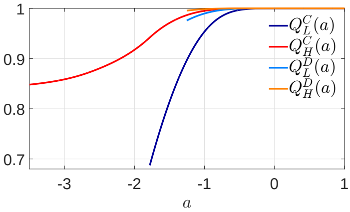
- Banks take deposits from net savers and buy debt from borrowers
- Debts are marketable
- Bonds pay a coupon equal to the risk free rate
- Macaulay duration of the bond is given by $1/\delta_A$

Debts are priced as bonds according to a PDE. Prices are denoted Q_i^C and Q_i^D, $i = L, H$

$$Q_i^C(a) = \frac{(r+\delta_A) + \frac{\partial Q_i(a)^C}{\partial a} S_i^C(a) + \lambda_{ij} Q_j^C(a)}{r + \delta_A + \lambda_{ij}}$$

$$Q_i^C = \frac{(r+\delta_A) + \lambda_{ij} Q_j^C + \lambda_i^{def} \phi_A Q_i^D(\phi_A a^*)}{r + \delta_A + \lambda_{ij} + \lambda_i^{def}}$$

$$\bar{Q}^C = \frac{(r+\delta_A) + \lambda_L^{def} \phi_A Q_L^D(\phi_A a^*)}{r + \delta_A + \lambda_L^{def}}$$



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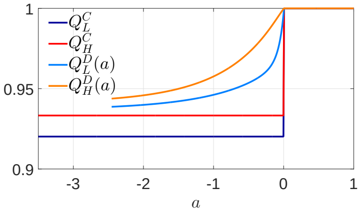

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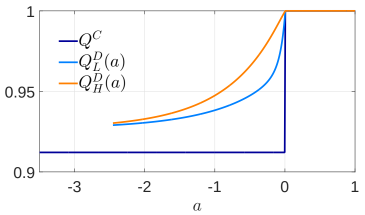

Recipe for equilibrium

- 4 Value functions for households
- F.O.C.s
- 2 Value functions for Q_i^D
- Value function for Q^C (1 or 2)
- $\sum_{i=1}^{4} \int_{\underline{a}}^{a_{max}} g_i(a) \mathrm{d}a = 1$
- Value matching* $\rightarrow V_L^C(a^*) = V_L^D(\phi_A a^*) - \psi_D$
- Smooth pasting** $\rightarrow V_L^{C\prime}(a^*) = \phi_A V_L^{D\prime}(\phi_A a^*)$

And we require that excess demand equals zero.

$$ED(r) = \sum_{i=1}^{4} \int_{\underline{a}}^{a_{max}} [z_i - c_i] g_i(a) \mathrm{d}a = 0$$

Numerical method: using finite differences as in Achdou et al. (2017) and Nuno and Thomas (2017) + LCP

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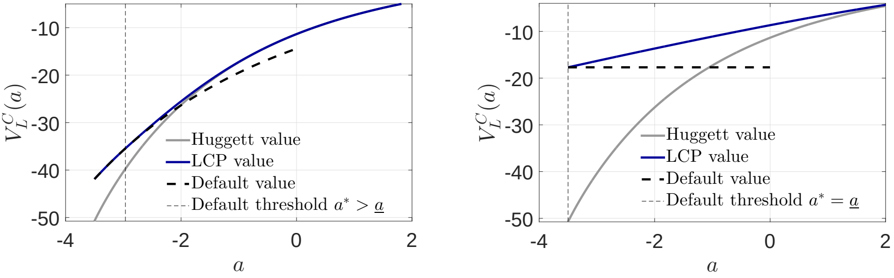
Choosing bankruptcy

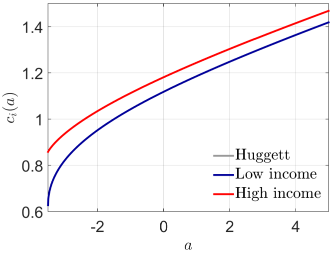
Default occurs when:

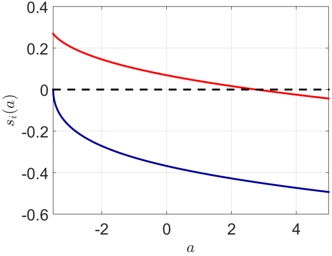
- Value matching is satisfied
- Happens in the interior if smooth pasting is satisfied



Consumption policy with bankruptcy

No information and limited information cases

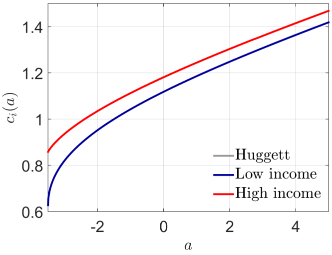


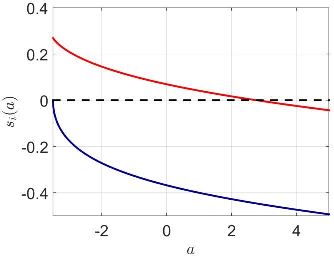


Wälde (2010) - Twin problem of HJB

Consumption policy with bankruptcy

Full information case

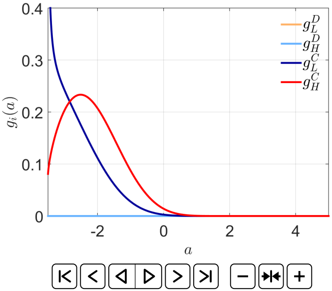


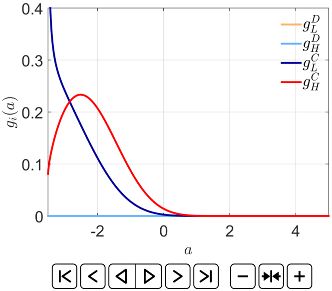


What happens with full information when r, δ_A change?

Wealth distribution with bankruptcy

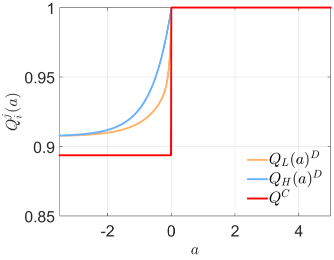
NI and FI

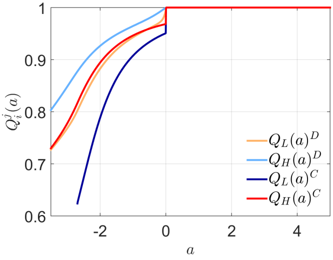




Bond prices with bankruptcy

NI and FI - GE results with $\delta_A \in [0 \quad 0.1035]$.





Consumption growth within state

- Bayer-Wälde (2010) / Achdou et al. (2017)

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\sigma}\left(r - \rho - \lambda_i \left[1 - \frac{u_j'(a)Q_j}{u_i'(a)Q_i}\right]\right)$$

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- This paper

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\sigma}\left(r_i(a) - \rho - \delta_A \frac{\partial Q_i}{\partial a}\frac{a}{Q_i} - \lambda_i\left[1 - \frac{u_j'(a)Q_j}{u_i'(a)Q_i}\right]\right)$$

where $r_i(a)$ represents the risk premium.

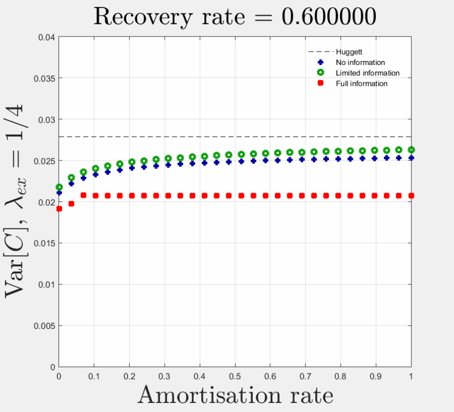
Outline

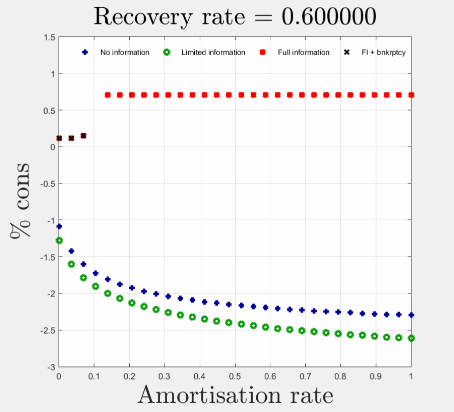
Calibration - sensitivity analysis in red

	Values	Description
σ	2	CRRA
λ_{LH}	0.25	Poisson rate $z_L \rightarrow z_H$
λ_{HL}	0.25	Poisson rate $z_H \rightarrow z_L$
z_L	0.75	Income of low type
z_H	1.25	Income of high type
ρ	0.056	Discount rate
λ_{ex}	[0.1 0.33]	Poisson out of exclusion
$\underline{a}$	-3.5	Exogenous debt limit
ψ_D	0.5	Mental cost of default
δ_A	[0,1]	Amortisation rate
ϕ_A	[0.5 0.9]	Recovery rate
ψ_1	$0.05{*}z_i$	Repayment plan
ψ_2	0.1	Repayment plan
ψ_3	1	Repayment plan

Bankruptcy + more info = higher welfare??

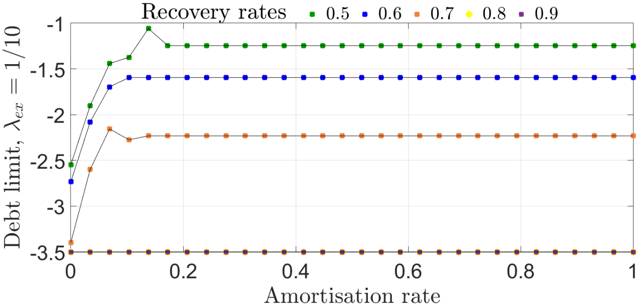
FI yields higher welfare than **LI** and **NI**. There is more lending but also more variance with less info. Maturity of debt has an impact on welfare.





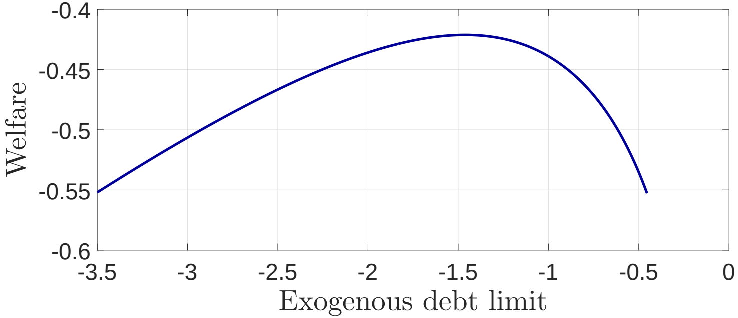
How? → Endogenous debt limit

When banks have full information, the debt limit endogenously moves to the interior of the state space - $\underline{a} \rightarrow a^*$



Why? → Obiols (2009)

The results are mainly driven by the debt limit effect described by Obiols



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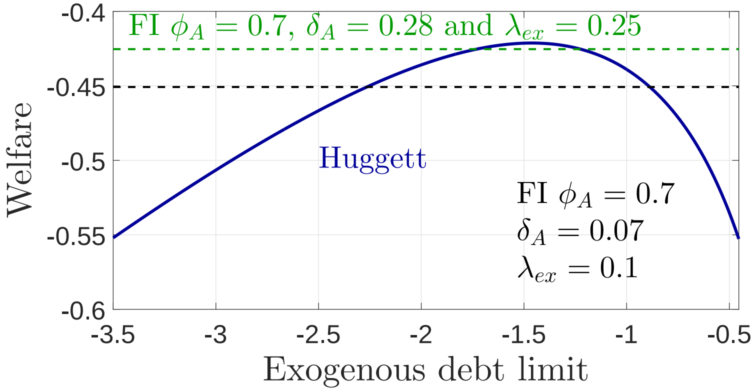

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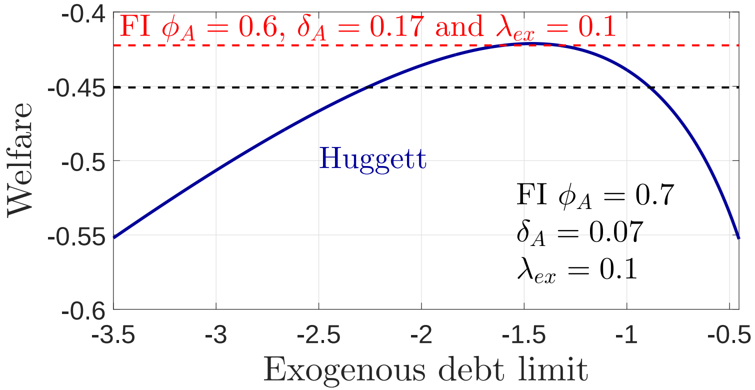

The effect of memory

If time allows... Click here

Outline

Conclusion gathered from preliminary results

- Bankruptcy increases risk sharing, $Var[C] \downarrow$
- However it penalises solvent borrowers through premiums
- The bank information set may morph premiums into rationing
- But tighter debt limits may improve welfare, even with $+$ve bankruptcy
- Moreover, *bankruptcy can take us to the optimal debt limit*
- Shorter maturities reduce W in NI and LI, raise W in FI
- There is more lending with less memory and less information
- LI is worse for all memories

Equilibrium interest rate

For an economy with preferences represented by a CRRA utility function, the equilibrium interest rate satisfies

- Representative agent - Achdou et al. (2017)

$$r^* = \rho - \frac{[v_1'(\underline{a}) - v_2'(\underline{a})]}{EMU} \lambda_{LH} M_1$$

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$$r^* = \rho - \frac{[v_1'(\underline{a}) - v_2'(\underline{a})]}{EMU} \lambda_{LH} M_1$$

- Here

$$r^* = \rho - \kappa M_3 - \frac{\text{ERP} + \delta_A \Gamma}{\text{EMU}}$$

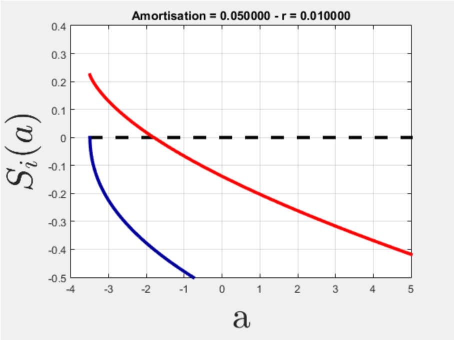
$$\kappa = \frac{\left([v_1'(a^*) - v_2'(a^*)] + [v_1'(a^*) - v_3'(\phi_A a^*)] \right) \lambda_{ex} + [v_3'(.)(\lambda_{ex} + \lambda_{LH}) - v_4'(.)\lambda_{LH}]}{EMU}$$

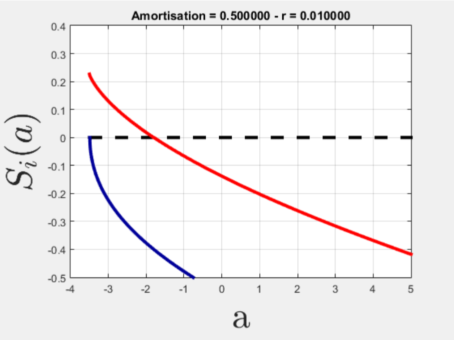
$$\Gamma = \sum_{i=1}^{4} \int_{a^*}^{\infty} \frac{\partial Q_i}{\partial a} a g_i(a) \mathrm{d}a$$

The formula captures all informational regimes and collapses to the Huggett HACT equation in Proposition 4 in Achdou et al. (2017)!

Drift, interest and amortisation under FI

As δ_A and $r \uparrow$ the low income agent is more likely to file for bankruptcy and raise consumption (Lagrange multiplier V_L^C falls)





But if banks have FI then the debt limit tightens!

Get back

Twin solution of continuous-time Bellman equation

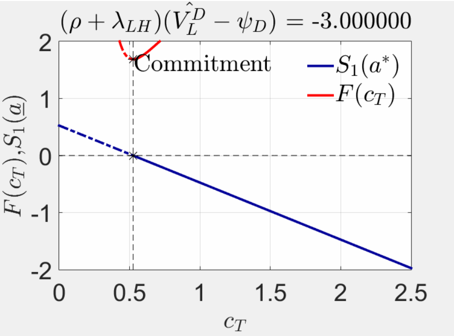
Table: Boundary Conditions

	$A_* \geq \underline{A}$	Value matching	Smooth pasting	Bankruptcy	Drift at A_*
i	not binding	holds at $A_* > \underline{A}$	holds at $A_* > \underline{A}$	at $A_* > \underline{A}$	–
ii	binding	holds at $A_* = \underline{A}$	does not hold	at $A_* = \underline{A}$	-
iii	binding	doesn't, $V(\underline{A}) > V_{def}(\phi_A(\underline{A}))$	no	no	0
iv	not binding	doesn't, $V(\underline{A}) > V_{def}(\phi_A(\underline{A}))$	no	no	+

Vlaue matching yields two roots:

- One implies $\dot{a} > 0$
- Bankruptcy implies $\dot{a} < 0$
- If $F(CT) > 0$ then $\dot{a} = 0$

Get back



- Two HJBs for bankruptcy

$$\rho V_i^D(a) = \max_c \; u(c) + \frac{\partial V_i^D(a)}{\partial a} S_i^D(a) + \lambda_i \left[V_j^D(a) - V_i^D(a) \right] + \lambda_{ex} \left[V_i^C(a) - V_i^D(a) \right]$$

- HJB for high income in commitment

$$\rho V_H^C(a) = \max_c \; u(c) + \frac{\partial V_H^C(a)}{\partial a} S_H^C(a) + \lambda_{HL} \left[V_L^C(a) - V_H^C(a) \right]$$

- HJBVI for low income in commitment

$$\min \left\{ \rho V_L^C(a) - \; u(c) - \boldsymbol{A} V_L^C(a), V_L^C(a) - [V_L^D(a) - \psi_D] \right\} = 0$$

where $\boldsymbol{A} V_L^C(a) = \dfrac{\partial V_L^C(a)}{\partial a} S_L^C(a) + \lambda_{LH} \left[V_H^C(a) - V_L^C(a) \right]$.

We set the time partials to zero... $\dfrac{\partial V_i^k(a)}{\partial t} = 0$

Get back

- KFE low income in commitment

$$\frac{\partial_t g_L^C(a,t)}{\partial t} = -\partial_a[s_L^C(a,t)g_L^C(a,t)] - |sg^*|\delta(a-a^*) + \lambda_{ex} g_L^D(a,t) + \lambda_{HL} g_H^C(a,t) - \lambda_{LH} g_L^C(a,t)$$

- KFE high income in commitment

$$\frac{\partial_t g_H^C(a,t)}{\partial t} = -\partial_a[s_H^C(a,t)g_H^C(a,t)] + \lambda_{ex} g_H^D(a,t) + \lambda_{LH} g_L^C(a,t) - \lambda_{HL} g_H^C(a,t)$$

- KFE low income bankrupt

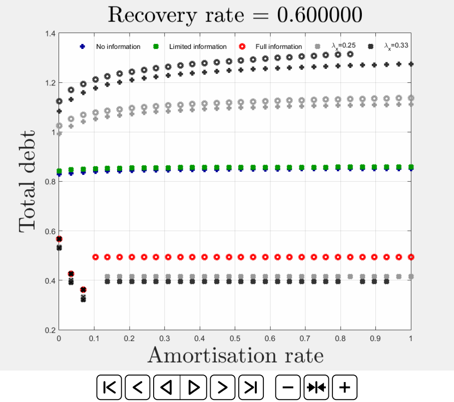
$$\frac{\partial_t g_L^D(a,t)}{\partial t} = -\partial_a[s_L^D(a,t)g_L^D(a,t)] + |sg^*|\delta(a-a^*) - [\lambda_{ex} + \lambda_{LH}]g_L^D(a,t) + \lambda_{HL} g_H^D(a,t)$$

- KFE High income bankrupt

$$\frac{\partial_t g_H^D(a,t)}{\partial t} = -\partial_a[s_H^D(a,t)g_H^D(a,t)] - [\lambda_{ex} + \lambda_{HL}]g_H^D(a,t) + \lambda_{LH} g_L^D(a,t)$$

where sg^* is the flow, at a^*, of low income commitment agents into bankruptcy. Get back

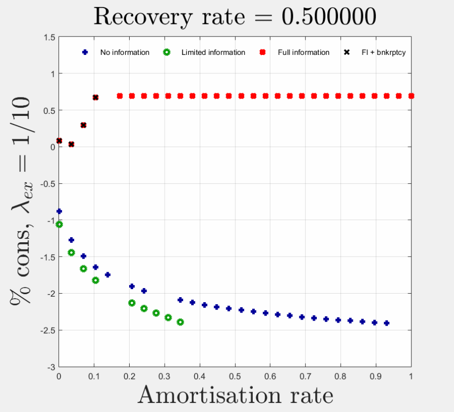
The effect of memory on CEL, Var[C] and TL

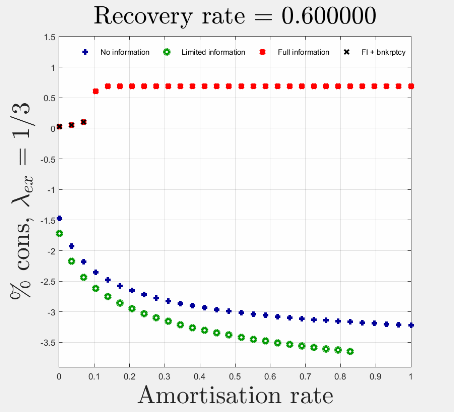


Get back

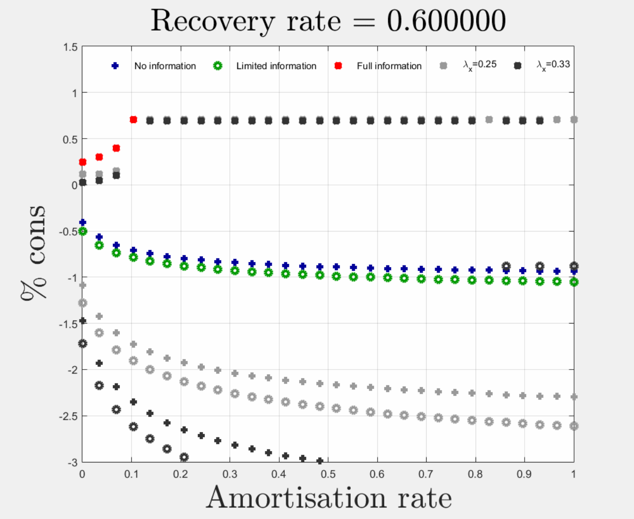
As pointed out by Elul and Gottardi (2015) there is more lending with less memory, but this only works when lenders do not have full information.

The effect of memory - CEL as $\lambda_{ex} = 1/10$ and $1/3$





The effect of memory, all regimes compared



Risk sharing - Var[C] as $\lambda_{ex} = 1/10$ and $1/3$

Bankruptcy in FI does improve risk sharing, but at the cost of higher premiums